α-MATHEMATICS

Grade 11 **Time: $2\frac{1}{2}$ hours**

Examiner: Pieter van Onselen **Total: 150 marks**

Moderator: Anna Muller

INSTRUCTIONS AND INFORMATION

Read through the following instructions before answering the question paper.

1. This question paper consists of 8 pages and an answer sheet of 2 pages.
2. Answer ALL 7 questions.
3. Number the answers according to the numbering system used in this question paper.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers must be given correct to two decimal places, where necessary.
6. Clearly show all calculations, diagrams, graphs etc. that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are Given in radians. Answers must also be Given in radians where necessary.
10. A formula sheet is included at the andd of this question paper.
11. Write neatly and legibly.

Question 1 [20 marks]

This question must be answered on the answer sheet.

Every question has **ONLY** one correct answer for **TWO** marks each. Mark the correct answer with an X on the answer sheet.

1.1 The radius and the angle of a sector with area A are doubled.

The area of the new sector is:

(A) $8A$

(B) $\frac{A}{2}$

(C) $2A$s

(D) $4A$

1.2 Given: $\frac{7-x}{(x+1)(x-3)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$

(A) $A = 1$

(B) $A = -2$

(C) $A = -1$

(D) $A = 2$

1.3 The inverse of $f(x) = \sin(2x) - \frac{\pi}{2}$ is:

(A) $y = \frac{sin\left(x+\frac{\pi}{2}\right)}{2}$

(B) $y = 2arcsin(x + \frac{\pi}{2})$

(C) $f^{-1}(x) = \frac{1}{2}arcsin(x + \frac{\pi}{2})$

(D) $f^{-1}(x) = \frac{1}{2}arcsin(x) + \frac{\pi}{2}$

1.4 The third term of the expansion $(x + b)^7$ is $84x^5$. Determine the value(s) of b:

(I) $b = 2$ (II) $b = -2$

(A) I **(B)** II **(C)** I or II **(D)** No answer

1.5 Given $y = \frac{1}{4}\arcsin(x+1)$.

If y is scaled vertically with a factor of 2, and translated 3 units to the left. The equation will be:

(A) $y = \frac{1}{2}\arcsin(x-2)$

(B) $y = \frac{1}{8}\arcsin(x+4)$

(C) $y = \frac{1}{8}\arcsin(x-2)$

(D) $y = \frac{1}{2}\arcsin(x+4)$

1.6 Which of the following statements are true?

(I) $|x| = \sqrt{x^2}$ (II) $|x| = \left(\sqrt{x}\right)^2$

(III) $|x| = |-x|$ (IV) $|x| > 0$

(A) (I) , (II) and (IV) **(B)** (I) , (II) and (III)

(C) (I) and (IV) **(D)** (I) and (III)

1.7 Determine the x-intercept of $y = arctan(x) + \frac{\pi}{4}$ where $y \in (-\frac{\pi}{2}; \frac{\pi}{2})$:

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4}$

(C) 1

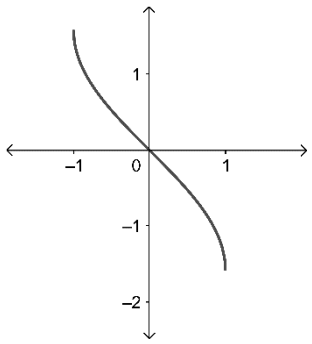
(D) -1

1.8 A circle with a radius of 6 cm is divided into 3 equal sectors. The arc length of the sector is ...

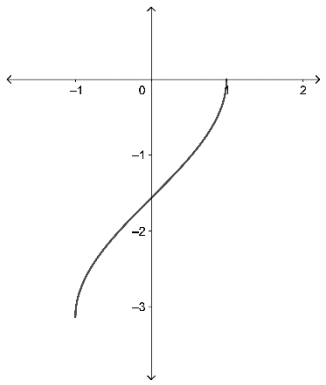
(A) $\frac{\pi}{3}$ cm **(B)** 4π cm **(C)** $\frac{2\pi}{3}$ cm **(D)** 2π cm

1.9 The graph of $y = -arccos(x) + \frac{\pi}{2}$ is:

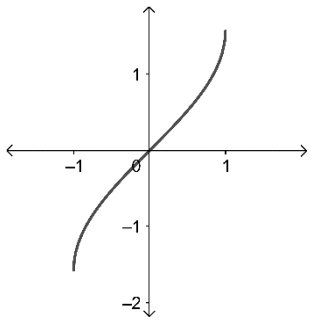
(A)



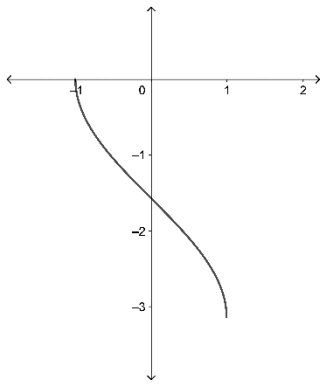
(B)



(C)



(D)



1.10 Determine the value of the 6th entry in the 10th row of Pascal's triangle.

(A) 210 **(B)** 252 **(C)** 84 **(D)** 126

Question 2 [35 punte]

2.1 Solve for x:

(a) $-|x-1|+8=0$ (4)

(b) $|x+1|<3$ (3)

2.2 Given $f(x)=|x+1|-2$ and $g(x)=2x$.

(a) Determine the coordinates of the intersection of $f(x)$ and $g(x)$. (4)

(b) Sketch the graph of $f(x)$ and $g(x)$. Use DIAGRAMSHEET 1 for the sketch. (6)

2.3 Use the ANTWER SHEET and answer the following questions. Match COLUMN B with COLUMN A. Note that COLUMN B may be used in more than one answer. (10)

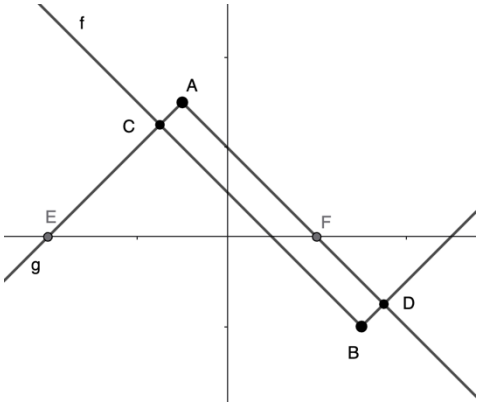
COLUMN A		COLUMN B	
2.3.1	$\|x-3\| \geq 0$	**A**	$x \geq 3$
2.3.2	$0 < \|x-3\|$	**B**	$6 \geq x \geq 0$
2.3.3	$\|x-3\| = 0$	**C**	No solution for $x \in \mathbb{R}$
2.3.4	$\|x-3\| \geq -3$	**D**	$x \in \mathbb{R}$
2.3.5	$-\|x-3\| \geq 0$	**E**	$x \geq 0$ or $x \leq 6$
		F	$x \in \mathbb{R}, x \neq 3$
		G	$x = 3$

2.4 In the diagram the graph of $f(x)=|x-a|+b$ and the graph $g(x)=-|x-c|+d$ is shown. (8)

The points $A\ (-1;y)$ and $B\ (3;-2)$ are given.

(a) Determine, with reasons, the values of a and b. (3)

(b) If $C\left(-\frac{3}{2};\frac{5}{2}\right)$ is the intersection of $f(x)$ and $g(x)$, Determine c and d. (5)



Question 3 [18 punte]

3.1 Decompose $\frac{x^3+2x^2+5x+2}{(x+1)^2(x^2+1)}$ into partial fractions. (10)

3.2 Use the ANSWER SHEET and answer the following questions. (8)

Match COLUMN B with COLUMN A. Note that COLUMN B may be used in more than one answer.

COLUMN **A**		COLUMN **B**	
3.2.1	$\frac{f(x)}{x^3+x}$	**A**	$\frac{A}{x^2}+\frac{B}{x+1}$
3.2.2	$\frac{f(x)}{x^4+x^2}$	**B**	$\frac{A}{x^2}+\frac{B}{x}+\frac{C}{x+1}$
3.2.3	$\frac{f(x)}{x^2(x+1)}$	**C**	$\frac{A}{x+1}+\frac{B}{x-1}$
3.2.4	$\frac{f(x)}{-x^2+1}$	**D**	$\frac{A}{x^2}+\frac{Bx+C}{x^2+1}$
		E	$\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$
		F	$\frac{A}{x}+\frac{Bx+C}{x^2+1}$
		G	$\frac{A}{x+1}-\frac{B}{x-1}$

Question 4 [14 punte]

4.1 Given $f(x) = 2x^3 - 3x^2 + 18x + 10$ and $x - 1 + 3i$ is one of the complex factors.

(a) Factorise $f(x)$ completely in $\mathbb{Q}[x]$. (7)

(b) Determine the solution of $f(x) = 0$ in $\mathbb{C}[x]$. (2)

4.2 Given $h(\sqrt{2}) = 0$ and $h(x) = x^4 - x^3 + ax^2 + 2x - a$.

(a) Determine the other irrational factor. (1)

(b) Subsequently determine the value of a. (4)

Question 5 [27 punte]

5.1 Given: $1 + 3 + 3^2 + 3^3 + \cdots + 3^{n-1} = \frac{3^n - 1}{2}$

(a) Give the sigma notation of $1 + 3^2 + 3^3 + \cdots + 3^{n-1}$. (2)

(b) Proof the statement with mathematical induction for $n \in \mathbb{N}$. (11)

5.2 Determine the coefficient of x^5 in the expansion $\left(x^2 - \frac{2}{x}\right)^{10}$. (6)

5.3 Given $\frac{1}{(4-x)^2}$.

(a) Determine the first 3 terms of $\frac{1}{(4-x)^2}$. (6)

(b) Determine the values of x for which the series converges. (2)

Question 6 [16 punte]

6.1 Given: $f(x) = arccos\left(\frac{x}{2}\right) + \frac{\pi}{3}$

(a) Give the domain and range of $f(x)$. (4)

(b) Sketch $f(x)$. (4)

Show all intercepts with the axes and the asymptotes of the sketch.

Use **DIAGRAM SHEET 2** for the sketch.

(c) Determine $f^{-1}(x)$ the inverse of $f(x)$. (4)

6.2 Solve for x : (4)

$$2sin\left(2x - \frac{\pi}{6}\right) = 1$$

Question 7 [20 punte]

7.1 A sector has radius r cm and arc length of s cm.

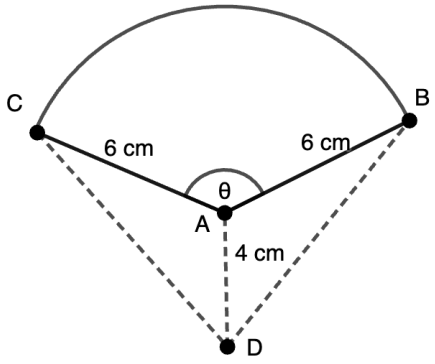
The arc length is given as $s = 2r + 4$. The area of the sector is $35\ cm^2$.

(a) Give the angle θ in terms of r and simplify your answer. (3)

(b) Show that the area can be given as $A = r^2 + 2r$. (3)

(c) The area of the sector is $35\ cm^2$. Determine the value of r. (4)

7.2 The sketch shows DCB forming a logo. The sector ACB with $\theta = 2.2\ radians$ and radius $AB = 6$ cm. The straight lines DC and DB are equal. The straight line $AD = 4$ cm.



(a) Determine the area of sector ABC. (2)

(b) Determine the angle $D\hat{A}C$. (3)

(c) Determine the area of enclosed figure in DCB. (5)

- END OF THE PAPER -

ALPHA MATHEMATICS FORMULA SHEET

ALGEBRA:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r$$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \text{ ; mits / } \textit{if } |x| < 1$$

MATRICES AND VECTORS:

Cramer's law: $x_i = \frac{|A_i|}{|A|}$

$\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$

$\boldsymbol{a} \bullet \boldsymbol{b} = a_x b_x + a_y b_y$

CALCULUS:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$\int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

TRIGONOMETRY:

In a sector: $s = r\theta$ and $A = \frac{1}{2}r^2\theta$

Triangle: $A = \frac{1}{2}absinC$

TABLE OF DERIVATIVES:

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$f[g(x)]$	$f'[g(x)].g'(x)$

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ANSWER SHEET

Name and surname:

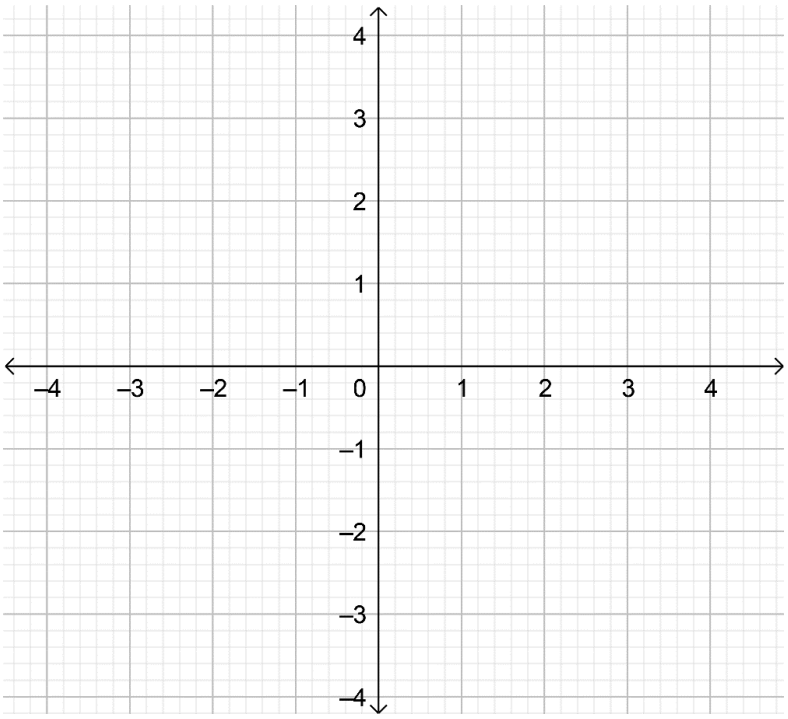
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Question Total	1 [20]	2 [35]	3 [18]	4 [14]	5 [27]	6 [16]	7 [20]	TOTAL 150
Learner mark								

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM SHEET 1 Question 2.2 (b)



Question 2.3

COLUMN A	COLUMN B
2.3.1	
2.3.2	
2.3.3	
2.3.4	
2.3.5	

Question 3.2

KOLOM A	KOLOM B
3.2.1	
3.2.2	
3.2.3	
3.2.4	

DIAGRAM SHEEtB 2 Question 6.1

