VRAAG 1

1.1	**A**	**B**	**X**	**D**
1.2	**X**	**B**	**C**	**D**
1.3	**A**	**B**	**C**	**X**
1.4	**X**	**B**	**C**	**D**
1.5	**X**	**B**	**C**	**D**
1.6	**A**	**B**	**C**	**X**
1.7	**A**	**B**	**X**	**D**
1.8	**A**	**X**	**C**	**D**
1.9	**A**	**B**	**C**	**X**
1.10	**A**	**X**	**C**	**D**
1.11	**A**	**X**	**C**	**D**
1.12	**X**	**B**	**C**	**D**
1.13	**X**	**B**	**C**	**D**
1.14	**A**	**B**	**X**	**D**
1.15	**X**	**B**	**C**	**D**

Verduidelikings:

1.6 Twee lineêre vergelykings het oneindig baie oplossings indien hulle op dieselfde reguit lyn lê, indien die gradiënt en die afsnitte dieselfde is of as verhoudings tussen die koëffisiënte dieselfde is. . In $6x-2y=4$ is $y=3x-2$ en in $mx-y=3$ is $y=mx-3$. Dan is $\frac{1}{1}=\frac{3}{3}\neq\frac{2}{3}$.

1.7 Gebruik substitusie os differensieer die antwoorde. $\frac{d}{dx}\left(-\frac{1}{4}\cos(2x)\right)+k=-\left(-\frac{1}{4}\sin 2x\right).2$ $=\frac{1}{2}\sin(2x)=\frac{1}{2},2\sin x\cos x.$ 1.8 Bereken bgcos(cos$\frac{4\pi}{3}$) op sakrekenaar (radiale)

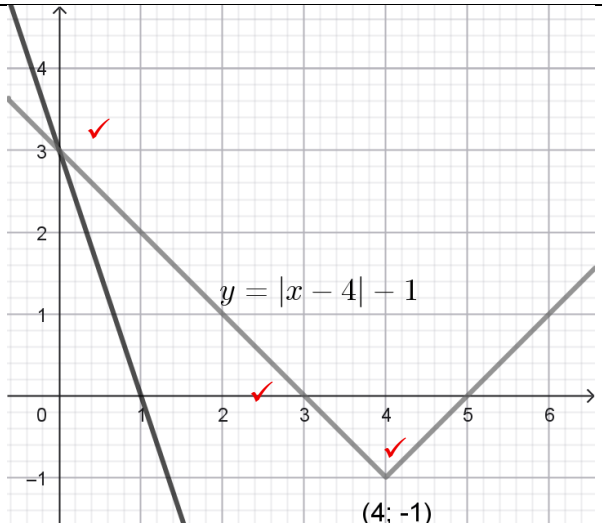
1.9 Gebruik kruisproduk. 1.10 $x=2$ gee $y=1$, $x<2$ gee $y=-1$, $x>2$ gee $y=-1$. Verwyderbaar.

1.12 Bereken y: $y=-y+1+1, y=1$. Diff: $y'=y+xy'+2x$ Vervang: $y'=1-y'-2; 2y'=-1$

1.13 Diff antwoorde 1.14 Diff antwoorde

1.15 Bepaal $\frac{dy}{dx}$ en verang $x=0$: Kwos reël. $\frac{dy}{dx}=\frac{\frac{1}{3}(8)^{-\frac{2}{3}}.0..(1)\text{^}*1/--\frac{1}{4}(1)^{-\frac{3}{4}}.2.(8)^{\frac{1}{3}}}{\sqrt{1}}=-1$

VRAAG 2 **[23 punte]**

2.1(a)	$f(2) = \frac{A}{2} + \ln 2^{-0,5} = 0,1534$ ✓ $A = 2(0,1534$ ✓ $- \ln 2^{-0,5}) = 2(0,1534 + 0,34657\ldots)$ $= 0,99995$ ✓	[3]
2.1(b)	$f'(t) = -\frac{1}{t^2}$ ✓ $+ \left(\frac{1}{t^{-0,5}}\right)$ ✓ $\times (0,5$ ✓$)t^{-1,5}$ ✓ OF $-\frac{1}{t^2}$ ✓✓ $-\frac{1}{2t}$ ✓✓	[4]
2.2 (a)	$\ln(x-1) - \ln(3x+2) = 1$ $\ln\frac{x-1}{3x+2} = 1$ ✓ $\frac{x-1}{3x+2} = e$ ✓ $x - 1 = 3xe + 2e$ ✓ $x(1-3e) = 2e + 1$ $x = \frac{2e+1}{1-3e} = -0,90$ ✓	[4]
2.2 (b)	$\ln(x-1) = \ln(-1,9$ ✓$)$ OF Beperkinge vir $\ln x$: $\ln(3x+2) = \ln(-0,7$ ✓$)$. $x > 1$ en $x > -\frac{2}{3}$ ✓ **Slegs een nodig** $\therefore x > 1$ maar $x = -0,9$ ✓ Rede: $\ln x$ is ongedefinieer as $x < 0$ vir reële getalle. ✓	[3]
2.3(a)	$\lvert x-4\rvert - 1 = 3 - 3x$, dus $\lvert x-4\rvert = 4 - 3x$ $x - 4 \geq 0$; $x \geq 4$ of $x - 4 < 0$ ✓ $x - 4 = 4 - 3x$; ✓ dus as $x < 4$: $4x = 8$; $x = 2$ nvt ✓ $-x + 4 = 4 - 3x$ ✓ $2x = 0$; $x = 0$. Dus net $x = 0$ ✓	[5]
2.3(b)	 ✓ ✓ ✓ vorm ✓	[4]

VRAAG 3 **[22 PUNTE]**

3.1	$x+1=\frac{-\sqrt{6}}{2}$ ✓ $(x^2+2x+1)=\frac{3}{2}$ ✓ $2x^2+4x-1$ is 'n faktor✓ $P(x)=2x^4+4x^3-9x^2-16x+4$ $=(2x^2$✓$+4x$✓$-1)(x^2-4)$✓ $=(x+1-\frac{\sqrt{6}}{2})(x+1+\frac{\sqrt{6}}{2})(x-2)(x+2)$ ✓	[7]
3.2 (a)	$z_1=4\sqrt{2}$✓$\text{cis}\,(\frac{3\pi}{4})$✓ en $z_2=4$✓$\text{cis}\,(\frac{7\pi}{6})$✓ OF $z_2=4$✓$\text{cis}\,(\frac{-5\pi}{6})$✓	[4]
3.2 (b)	$\frac{(4\sqrt{2})^2 cis\frac{3\pi}{2}}{4^3 cis\frac{7\pi}{2}}$✓✓ $=\frac{1}{2}$✓$\text{cis}(-2\pi)$✓ $=\frac{1}{2}+0\,i$ ✓	[5]
3.3	$z^4=16\,\text{cis}\,0$ ✓ $\therefore z_k=(16\,\text{cis}\,0)^{\frac{1}{4}}$ ✓ $=16^{\frac{1}{4}}\,\text{cis}\,\frac{0+2\pi k}{4}$ $k\in\{0,1,2,3\}$ $z_1=2\,\text{cis}\,0$ ✓ $z_2=2\,\text{cis}\,\frac{\pi}{2}$ ✓ $z_3=2\,\text{cis}\,\pi$ ✓ $Z_4=2\,\text{cis}\,\frac{3\pi}{2}$ ✓	[6]

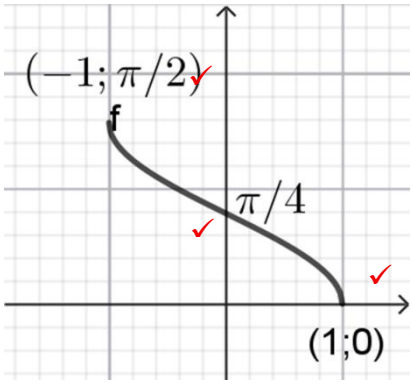
VRAAG 4 **[19 punte]**

4.1	1✓ $+\left(-\frac{1}{3}\right)(3x)$✓ $+\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{1}{2!}\right)(3x)^2$✓ $+$ $+\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(\frac{1}{3!}\right)(3x)^3$✓ $+\cdots$ $=1-x$✓ $+2x^2-\frac{14}{3}x$✓	[6]
4.2	$7_c0\left(\frac{x}{2}\right)^7$ ✓ $+7_c1\left(\frac{x}{2}\right)^6\left(-\frac{2}{x^2}\right)$ ✓ $+$ $7_c2\left(\frac{x}{2}\right)^5\left(-\frac{2}{x^2}\right)^2$ ✓ $+\cdots$ $=\frac{x^7}{128}$ ✓ $-\frac{7x^4}{32}$✓ $+\frac{21}{8}x$✓ $+\cdots$	[6]
4.3	Booglengte = $r\theta=6\times3,5=21$✓ $D\hat{A}C=2\pi-3,5-\frac{\pi}{2}=1,21$ rad✓ $DC^2=9^2+6^2-2(9)(6)\cos(1,21)$✓ $=78,87$ ✓ $\therefore DC=8,88$ ✓ Omtrek = $21+8,88+9+6$✓ $=44,88$ ✓ cm	[7]

VRAAG 5 **[13 punte]**

5.1(a)	$\mathbf{9} = \boldsymbol{k}.\mathbf{6}$; ✓ $\boldsymbol{k} = \frac{3}{2}$ ✓	[2]
5.1 (b)	ewewydig✓	[1]
5.1(c)	Grootte van v = $\sqrt{(\mathbf{36}+\mathbf{16}+\mathbf{4})} = \sqrt{\mathbf{56}} = \mathbf{2}\sqrt{\mathbf{14}}$✓ Dus eenheidsvektor = $\left(\frac{\mathbf{3}}{\sqrt{\mathbf{14}}}\boldsymbol{i}$✓ $-\frac{\mathbf{2}}{\sqrt{\mathbf{14}}}\boldsymbol{j}$✓ $+\frac{\mathbf{1}}{\sqrt{\mathbf{14}}}\boldsymbol{k}\right)$ ✓ $= \left(\frac{\mathbf{3}\sqrt{\mathbf{14}}}{\mathbf{14}}\boldsymbol{i} - \frac{\mathbf{2}\sqrt{\mathbf{14}}}{\mathbf{14}}\boldsymbol{j} + \frac{\sqrt{\mathbf{14}}}{\mathbf{14}}\boldsymbol{k}\right)$	[4]
5.2	Stel vektore is $\boldsymbol{u}_1 - \mathbf{2}\boldsymbol{u}_2 + \mathbf{3}\boldsymbol{u}_3$ en $\mathbf{3}\boldsymbol{v}_1 - \mathbf{2}\boldsymbol{v}_2 + \boldsymbol{v}_3$ Dan is $\boldsymbol{u}.\boldsymbol{v}$ = 3 + 4 + 3 = 10. ✓ Verder is $\lvert\boldsymbol{u}\rvert = \sqrt{\mathbf{1}+\mathbf{4}+\mathbf{9}} = \sqrt{\mathbf{14}}$✓ en $\lvert\boldsymbol{v}\rvert = \sqrt{\mathbf{9}+\mathbf{4}+\mathbf{1}} = \sqrt{\mathbf{14}}$✓ maar $\boldsymbol{u}.\boldsymbol{v} = \lvert\boldsymbol{u}\rvert.\lvert\boldsymbol{v}\rvert\ \mathbf{cos}\ \boldsymbol{\theta}$ met $\boldsymbol{\theta}$ die hoek tussen die vektore. Dus $\mathbf{10} = \sqrt{\mathbf{14}}\sqrt{\mathbf{14}}\,\mathbf{cos}\,\boldsymbol{\theta}$ ✓ $\boldsymbol{\theta} = \mathbf{bgcos}\ \frac{\mathbf{10}}{\mathbf{14}}$✓ $= \mathbf{0,78}$ ✓	[6]

VRAAG 6 **[18 punte]**

6.1 (a)	(a) Beperk $x \in [0;\frac{\pi}{2}]$. ✓	[2]
6.1 (b)	 vorm ✓	[4]
6.2(a)	$\theta = \mathbf{bgtan}(-1)$. $\theta = -\frac{\pi}{2}$ ✓	[1]
6.2 (b)	$\sin\frac{\theta}{2} = -1$ $\frac{\theta}{2} = \mathbf{bgsin}(-1) + 2\pi k$ met $k \in \mathbb{Z}$ ✓ $\frac{\theta}{2} = -\frac{\pi}{2} + 2\pi k$ ✓ $\theta = \pi + 4\pi k$ ✓ met $k \in \mathbb{Z}$	[3]
6.3	Stel $n = 1$: LK = 13 RK $= \frac{33-7}{2} = 13$ ✓ Dus is die bewering is waar vir $n = 1$. Aanvaar ✓ die bewering is waar vir $n = k$: $13 + 6 - 1 + \ldots + (20 - 7k) = \frac{33k-7k^2}{2}$ ✓ Te bewys vir $n = k + 1$: RK $= \frac{33(k+1)-7(k+1)^2}{2}$ ✓ $= \frac{33k+33-7k^2-14k-7}{3} = \frac{-7k^2+19k+26}{2}$ ✓ OF $= \frac{(k+1((33-7)}{2}$ en vir LK ook. LK $= \frac{33k-7k^2}{2}$ ✓ $+ 20 - 7k - 7$ ✓ $= \frac{33k-7k^2+40-14k-14}{2} = \frac{-7k^2+-19k+26}{2}$ ✓ Dus indien bewering waar is vir $n = k$ is dit ook waar vir $n = k + 1$. Volgens die beginsel van wiskundige induksie, is bewering waar vir alle $n \in \mathbb{N}$. -1 indien laaste bewoording nie daar is nie.	[8]

VRAAG 7 **[21 punte]**

7.1(a)	$\frac{1}{\cot 4x^2 \checkmark .\ln 10 \checkmark}(-\text{cosec}^2 4x^2)\checkmark(8x)\checkmark$	[4]
7.1 (b)	$g'(x) = -\sin\left(\frac{x}{5}\right)\checkmark .\frac{1}{5}\checkmark .\text{bgcos}\sqrt{5x}\checkmark + \cos(\frac{x}{5}).\frac{-1}{\sqrt{1-5x}}\checkmark \times \frac{\sqrt{5}}{2\sqrt{x}}\checkmark$	[5]
7.2	$2xy\checkmark + x^2.y'\checkmark + \frac{1}{y}.y'\checkmark = 2x$ $y'\left(x^2+\frac{1}{y}\right) = 2x - 2xy\checkmark$ $y' = \frac{2x-2xy}{x^2+\frac{1}{y}}\checkmark$	[5]
7.3(a)	$\frac{dy}{dx} = \frac{2e^{2x}(1+3x)\checkmark - 3(1+e^{2x})\checkmark}{(1+3x)^2} = 0$ $2e^{2x} + 6xe^{2x} - 3 - 3e^{2x} = 0\checkmark$ $2 + 6x - \frac{3}{e^{2x}} - 3 = 0$ $6x = 1 + \frac{3}{e^{2x}}\checkmark$ $x = \frac{1}{6} + \frac{1}{2}e^{-2x}\checkmark$ $a = \frac{1}{6}\ ;\ b = \frac{1}{2}\ \therefore a+b = \frac{2}{3}\checkmark$	[6]
7.3(b)	Newton se metode✓ Of Newton-Raphson se metode	[1]

8.1(a)	$x = -3$ ✓ , $x = 2$ ✓ en $x = 4$ ✓	[3]
8.1 (b)	$x = 2$: $f''(2) = -50 < 0$ Dus maksimum draaipunt. ✓ $x = 4$: $f''(4) = 98 > 0$ Dus minimum draaipunt. ✓ $x = -3$: $f''(-3) = 0$. $f''(-4) = -110$ en $f''(-2) = 38$. ✓ Dus buipunt, ✓ want tekens van 2e afgeleide weerskante verskil. ✓ **OF** Hellings kan van grafiek afgelees word $x = 2$: $f''(2) < 0$ Dus maksimum draaipunt. ✓ $x = 4$: $f''(4) > 0$ Dus minimum draaipunt. ✓ $x = -3$: $f''(-3) = 0$. $f''(-4) < 0$ en $f''(-2) > 0$. ✓ Dus buipunt, ✓ want tekens van 2e afgeleide weerskante verskil. ✓	[5]
8.2	b=$\Delta x = \frac{3}{n}$ ✓ en $x_i = -1 + \frac{3i}{n}$ ✓ $h = f(x_i) = 1 - \frac{6i}{n} + \frac{9i^2}{n^2} + m$ ✓ $\sum_{i=1}^{n} f(x_i) = \sum_{i=1}^{n}(1 + m - \frac{6i}{n} + \frac{9i^2}{n^2})$ ✓ $= n(1+m)$ ✓ $-\frac{6}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right)$ ✓ $+\frac{9}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)$ ✓ $= n + nm - 3n - 3 + 3n + \frac{9}{2} + \frac{3}{2n}$ $= n(1+m) + \frac{3}{2} + \frac{3}{2n}$ $\sum_{i=1}^{n} f(x_i)\,\Delta x = 3(1+m) + \frac{9}{2n} + \frac{9}{2n^2}$ ✓ Gegee: $\int_{-1}^{2}(x^2 + m)\,dx = 21$ $Dus \lim_{n\to\infty}(3(1+m) + \frac{9}{2n} + \frac{9}{2n^2}) = 21$. Dus $3(1+m) = 21$; $m = 6$ ✓	[9]

VRAAG 9 **[22 punte]**

9.1	$\int (2x^2 + 2 + \frac{2}{x})dx$ $= \frac{2x^3}{3}$ ✓ $+ 2x$ ✓ $- 2\ln x$ ✓ $+ k$ ✓	[4]
9.2	$\int \left(\sec^2 x - 1$ ✓ $+ \frac{1}{2}(1 + \cos(2x)]$ ✓$\right) dx$ $= \tan x$ ✓ $- x$ ✓ $+ \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)$ ✓ $+ k$ $= \tan x - \frac{x}{2} + \frac{1}{4}\sin 2x + k$	[5]
9.3	: $\frac{3x^2+3x+7}{(x^2+1)(x-2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-2}$, ✓ Dus $3x^2 + 3x + 7 = (Ax + B)(x - 2) + C(x^2 + 1)$ Stel $x = 2$: $25 = 5Cx$; $dus\ C = 5$ ✓ x^2: $3 = A + C$, $dus\ A = -2$ ✓ Konst: $7 = -2B + C$; $dus\ B = -1$ ✓ Dus $\int \frac{-2x}{x^2+1}dx - \int \frac{1}{x^2+1}dx + \int \frac{5}{x-2}dx$ $= -\ln(x^2 + 1)$ ✓✓ $- bgtan x$ ✓ $+ 5\ln(x - 2)$ ✓ $+ k$	[8]
9.4	$\int (\text{cosec}^3 2\theta \cot 2\theta)\, d\theta$ Aangesien $\frac{d}{dx}\text{cosec}\,\theta = -\text{cosec}\,\theta . \cot\theta$ ✓ $\int (\text{cosec}^3 2\theta \cot 2\theta)\, d\theta =$ $-\int (\text{cosec}^2 2\theta)(-\text{cosec}\, 2\theta . \cot 2\theta)d\theta$ ✓ $= -\frac{cosec^3 2\theta}{3.2}$ ✓✓ $+ k$ **OF** Stel $u = \text{cosec}\, 2\theta \quad \therefore \frac{du}{d\theta} = -\text{cosec}\, 2\theta . \cot 2\theta . 2$ $\int \frac{u^3}{-2u}du = \frac{-1}{2}\int u^2 du = -\frac{u^3}{6} + k$	[5]

$\int (\text{cosec}^3 2\theta \cot 2\theta)\, d\theta$ **Sonder substitusie:** $= \int (\text{cosec}^2 2\theta\, (\text{cosec}\, 2\theta \cot 2\theta)$ ✓$)\, d\theta$ $= \frac{-\text{✓cosec}^3\text{✓}2\theta}{3\text{✓}.2\text{✓}} + k$	Met substitusie: $= \int (\text{cosec}^2 2\theta\, (\text{cosec}\, 2\theta \cot 2\theta)$ ✓$)\, d\theta$ Stel $u = \text{cosec} 2\theta$ ✓ $\therefore \frac{du}{d\theta} = -2\text{cosec}\, 2\theta . \cot 2\theta$ ✓ Dan $\int (\text{cosec}^2 2\theta\, (\text{cosec}\, 2\theta \cot 2\theta))\, d\theta = -\frac{\int u^2 du}{2}$ ✓ $= -\frac{u^3}{3.2} + k = \frac{-\text{cosec}^3 2\theta}{3.2} + k$ ✓

$\int (\text{cosec}^3 2\theta \cot 2\theta)\, d\theta$

VRAAG 10 **[15 punte]**

10.1 (a)	$f(x) = \text{bgtan}(2x)$; $f'(x) = \frac{2}{1+4x^2}$ ✓✓	(2)
(b)	Stel $h(x) = \text{bgtan}\,2x - x + 2$ ✓ Dan $h'(x) = \frac{2}{1+4x^2} - 1$ ✓ Stel $x_1 = x_0 - \frac{\text{bgtan}\,2x_0 - x_0 + 2}{\frac{2}{1+4x_0^2} - 1}$ ✓ $x_1 = 3{,}4259$ ✓ Reg afgerond ✓	(5)
10.2	$\int_0^{\frac{\sqrt{3}}{2}} x.\,\text{bgtan}2x\,dx$ $Stel\ f = \text{bgtan}2x\ en\ g' = x$ ✓ $f' = \frac{2}{(1+4x^2)}\ en\ \ g = \frac{1}{2}x^2$ ✓ $Dus\ \int x.\,\text{bgtan}2x\,dx = \frac{1}{2}x^2.\,\text{bgtan}2x - \frac{1}{4}\int \frac{4x^2}{1+4x^2}dx$ ✓ $\therefore \int x\,\text{bgtan}2x\,dx$ $= \frac{1}{2}x^2.\,\text{bgtan}2x - \frac{1}{4}\int (1 - \frac{1}{1+4x^2})dx$ ✓ $= \frac{1}{2}x^2.\,\text{bgtan}2x - \frac{x}{4} + \frac{1}{8}\text{bgtan}2x \Big|_0^{\frac{\sqrt{3}}{2}}$ ✓ $= \frac{3}{8}.\frac{\pi}{3} - \frac{\sqrt{3}}{8} + \frac{1}{8}.\frac{\pi}{3} - 0$ ✓ $= \frac{3\pi}{8.3} + \frac{1}{8}.\frac{\pi}{3} - \frac{\sqrt{3}}{8} = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$ ✓ $\therefore a = \frac{1}{6}\ \ en\ \ b = -\frac{1}{8}$ ✓	[8]